Title: Going In, Out, and Around with Polygons

Brief Overview:

Students will construct, inscribe, and circumscribe polygons ranging from three to eight sides. Using area and perimeter measurements they will reach conclusions that will lead to the meaning of a limit.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will determine the best way to construct an inscribed or circumscribed polygon.

- **Mathematics as Reasoning**
  Students will reason to construct polygons.

- **Number and Number Relationships**
  Students will compare perimeters and areas of polygons and make conjectures based on observations.

- **Algebra**
  Students will use algebraic formulas and reasoning to explain their results.

- **Geometry**
  Students will realize the relationship between inscribed and circumscribed polygons as the number of sides increases.

- **Conceptual Underpinnings of Calculus**
  Students will be introduced to the concept of limit.

Grade/Level:

Grades 9-12

Prerequisite Knowledge:

Students should have working knowledge of the following:

- TI-92 calculator
- Cabri Geometry
- Determining ratios
- Concepts of perimeter, area, circumscribed polygons, and inscribed polygons

Objectives:

Students will:

- work cooperatively in groups.
- collect and organize data from the activity.
- evaluate a situation and give appropriate support for their answer.
- construct polygons using Cabri Geometry on the TI-92.
- calculate areas and perimeters of polygons.
- use ratios to compare areas and perimeters of polygons.
- develop the concept of limit.

Materials/Resources/Printed Materials:

- Paper and pencils
- TI-92 calculator or computer with current geometry software
- Student worksheets
- Teacher Resources

Development/Procedures:

- Review necessary constructions on the TI-92 using the **What Do You Remember** sheet. Begin by constructing a circle. Then, construct a polygon, without specifying whether it should be regular. Have students find and record the perimeter and area of the polygon. Next, construct a circumscribed polygon about the circle with the same number of sides as the original polygon. Calculate the perimeter and area of this polygon and then the ratio of the two perimeters and areas. Record these and complete worksheet.

- Divide class into pairs. Each student constructs a circle on his/her screen. One person will construct polygons with an odd number of sides and the other will construct polygons with an even number of sides. Students then will construct their first assigned polygon inscribed in and circumscribed about the circle on their screens. Using these polygons, students will calculate perimeter and area of their polygons and determine the ratio of the perimeters, areas, and ratio of these ratios. Record this data on the **What's the Size of It** sheet and complete the table for the assigned polygons (odd number or even number of sides).

- Pairs will share data to complete the table and answer questions on **What Are Your Thoughts** sheet.

- Class will discuss results and conjectures and will discover explanations for the relationships.

Performance Assessment:

The teacher will circulate around the room to observe groups dynamics. Students will be evaluated in the areas of participation, completion of assignments, and proficiency in using the TI-92 calculator.

Extension/Follow Up:

- Discover the relationship between the measure of the interior angle of a regular polygon and the number of sides.
- Construct regular polygons using rotations of a segment.
- Explain mathematically (algebraically) why the ratio of ratios is equal to the ratio of perimeters.
- Research limits.

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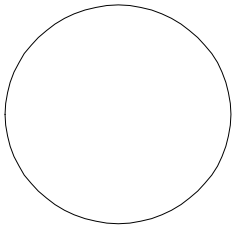
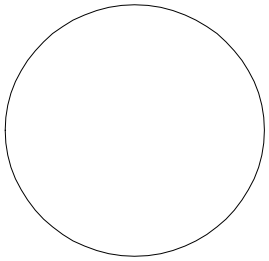
Name:_____
Date:_____

WHAT DO YOU REMEMBER???

1. Draw an inscribed polygon in the circle on the left. Draw a circumscribed polygon about the circle on the right.





2. On your TI-92 calculator, draw a circle on the left side of the screen. On the right side, construct a polygon. Draw below what is on your screen.

3. Using calculate on the TI-92, find the perimeter and area of the polygon and record your findings here.

   Perimeter_____
   Area_____

4. Construct a circumscribed polygon about the circle with the same number of sides as the polygon you've constructed. Draw below what is on your screen.

5. Using calculate on the TI-92, find the perimeter and area of this polygon, then find the ratio of the two perimeters and areas.

   Perimeter_____ Perimeter Ratio_____
   Area_____ Area Ratio_____

Name:_____
Date:_____

WHAT'S THE SIZE OF IT?

Complete the following table based on information you determine using the TI-92 calculator.

NUMBER OF SIDES	NAME OF POLYGON	$P_c/P_i = P_r$	$A_c/A_i = A_r$	A_r/P_r
3				
4				
5				
6				
7				
8				

P = Perimeter
A = Area
c = circumscribed
I = inscribed
r = ratio

Name:_____
Date:_____

What Are Your Thoughts?

Answer the following questions based on the information you have in your table.

1. Are the perimeter ratios the same or different?

2. Are the area ratios the same or different?

3. Do you notice any pattern in the perimeter ratios? Explain.

4. Do you notice any pattern in the area ratios? Explain.

5. Drag the circle to make it larger and smaller. What happens to the areas, perimeters, and ratios? Does this change your answer for number 3 and/or number four above? Explain.

6. What would you predict the ratios of the perimeters for a 20-sided polygon would be? The ratios of the areas? Explain.

7. How do you think the circumference of the circle would be related to the perimeters of the polygons? Explain.

8. How do you think the area of the circle would be related to the areas? Explain.

9. What do you notice about the ratio of ratios? Do they have anything in common with each other? With other ratios in the table? Explain.

FOR THE TEACHER: NOTES AND STROKES

I. **What Do You Remember** Sheet/Activity

1. Check students' work before moving on.

2a. Distribute calculators and have students turn them on. Press APPS, 8 for Geometry, and 3 for New. Go to variable and assign some arbitrary variable name, a (a temporary work area) and press ENTER twice. Next, press F3, 1 (circle) and ENTER. Move the cursor to locate the desired center of the circle. Press ENTER, and then use the cursor to select the radius and ENTER to draw the circle. If the circle is not already there, drag it (at the center point using the grab hand) to the left side of the screen.

2b. Select F3, 4 for Polygon. Move the cursor to the right and select the first vertex of the polygon. Using ENTER, continue to select vertices until you have the desired number of vertices/sides. Remember the last vertex must complete the polygon (this point), ENTER.

3a. To calculate perimeter, select F6, 1 for Distance and Length. Move the cursor to the polygon and you should see PERIMETER OF THIS POLYGON? Press ENTER.

3b. To calculate area, select F6, 2 for Area. Move the cursor to the polygon and you should see THIS POLYGON? Press ENTER.

4a. Move the cursor to the center of the circle. Press F2, 5 for Segment, and ENTER. Move the cursor to the circle to construct a radius, then ENTER. Repeat this to construct the same number of radii as sides in the polygon.

4b. Select F4, 1 for Perpendicular Line, then Enter. Move the cursor to one radius and you should see PERPENDICULAR TO THIS SEGMENT? After ENTER, move the cursor to the endpoint of this radius (on the circle) and you should see THRU THIS POINT? Selecting ENTER will create the tangent line. Repeat this for each radius.

4c. Select F2, 3 for Intersection Point, and ENTER. Move the cursor to the intersection of two of the tangent lines (that will become a vertex of the polygon) and you should see POINT AT THIS INTERSECTION? ENTER to create the point. Repeat this to create all vertices of the circumscribed polygon.

4d. Select F7, 1 for Hide/Show and ENTER. Move cursor to each tangent line, pressing ENTER at each , then move the cursor to each radius and press ENTER at each. Press ESC to hide these lines and segments.

4e. Select F3, 4 for Polygon and create the polygon by pressing ENTER at each vertex point. Remember the last vertex must complete the polygon (this point), ENTER. (A short-cut is to press ENTER twice after the last vertex).

5a. Follow directions for number 3 to determine perimeter and area for this new polygon.

5b. Select F6, 6 for Calculate, then ENTER. Using the cursor, move up so the perimeter of the circumscribed polygon is highlighted, ENTER. Press the ÷, move the cursor up so the perimeter of the other polygon is highlighted , ENTER. ENTER again to calculate this ratio.

5c. Repeat 5b using areas of the polygons.

II. **What's the Size of It** Sheet/Activity

1. Have the students divide into pairs and determine who will do the polygons with an odd number of sides and who will do the ones with an even number of sides. Then they should fill in the names for these polygons.
2. Students should begin with their first polygon (either 3-sided or 4-sided), drawing the circle first, then the inscribed polygon. This is done by selecting F3, 4 as in the previous activity (#2b) using points on the circle as the vertices. Now, construct the circumscribed polygon as in the previous activity (#4).
3. Determine the perimeters and areas of the polygons as in #3 of the previous activity, and then the ratios of these perimeters ($P_c/P_i = P_r$) and areas ($A_c/A_i = A_r$) as in #5.
4. Calculate the ratio of the two determined ratios (P_r and A_r) by moving the cursor up to the area ratios so it is highlighted (you should still be in Calculate) and ENTER. Then press ÷ and move the cursor up so the other ratio is highlighted, ENTER twice to obtain the new ratio. You may need to grab and drag (after pressing ESC to get to the pointer tool) if the numbers overlap each other.
5. This data should now be recorded in the table and students should repeat this for the other polygons.

III. **What Are Your Thoughts** Sheet/Activity

1,2. These ratios should not be the same.

3,4. The more sides the polygon has, the closer the ratio is to one. This is a good time to introduce the concept of limit to your students.

5. When the circle is dragged, the perimeters and areas change, but the ratios do not.
6. The ratios should be VERY close to 1.
7. The circumference of the circle should be between the perimeters of the pair of polygons for each n-gon. Upper and lower bounds could be introduced at this point.
8. The area of the circle should be between the areas of the pair of polygons for each n-gon. Upper and lower bounds could be introduced at this point.
9. The ratio of ratios is VERY close to the ratio of perimeters for each n-gon. As an extension, students could explain this algebraically. A starting point is to look at a regular polygon and then generalize from there for any polygon.

IV. **Extension** - How to create a regular polygon using rotations of a segment...

Using F2, 5 for Segment, ENTER, construct a segment of desired length keeping in mind that this is a side of a polygon (consider size and location). Press F7, 6 for Numerical Edit and ENTER. In the box input the measure of an interior angle of the polygon (this extension should be done after the first in the list) and ENTER. Select F5, 2 for Rotate, ENTER, and move the cursor to the segment (ROTATE THIS SEGMENT), ENTER, then move to one of the endpoints (AROUND THIS POINT), ENTER, then move to the angle measure (USING THIS ANGLE?), ENTER. Repeat this rotation process to complete the polygon.